Coimisiún na Scrúduithe Stáit
State Examinations Commission

Scéimeanna Marcála — Scrúduithe Ardteistiméireachta, 2007

Matamaitic Fheidhmeach — **Gnáthleibhéal**

Marking Scheme — Leaving Certificate Examination, 2007

Applied Mathematics — **Ordinary Level**

M31

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General Guidelines

1. Penalties of three types are applied to candidates' work as follows:

   Slips - numerical slips S(-1)

   Blunders - mathematical errors B(-3)

   Misreading - if not serious M(-1)

   Serious blunder or omission or misreading which oversimplifies:
   - award the attempt mark only.

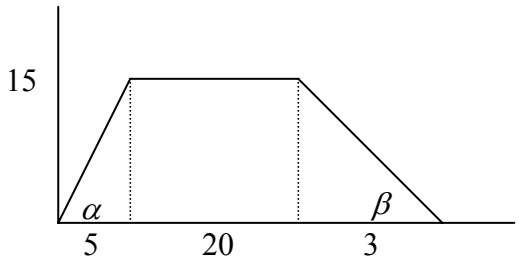
   Attempt marks are awarded as follows: 5 (att 2), 10 (att 3).

2. The marking scheme shows one correct solution to each question. In many cases there are other equally valid methods.

1. A car travels from p to q along a straight level road.
It starts from rest at p and accelerates uniformly for 5 seconds to a speed of 15 m/s.
It then moves at a constant speed of 15 m/s for 20 seconds.
Finally the car decelerates uniformly from 15 m/s to rest at q in 3 seconds.

(i) Draw a speed-time graph of the motion of the car from p to q.
(ii) Find the uniform acceleration of the car.
(iii) Find the uniform deceleration of the car.
(iv) Find $|pq|$, the distance from p to q.
(v) Find the speed of the car when it is 13.5 metres from p.

(i)



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(ii) $v = u + at$ $\qquad a = \tan\alpha$

$15 = 0 + 5a$ or $a = \dfrac{15}{5}$

$a = 3\ \text{m/s}^2$ $\qquad a = 3\ \text{m/s}^2$

10

(iii) $v = u + at$ $\qquad a = \tan\beta$

$0 = 15 + 3a$ or $a = \dfrac{15}{3}$

$a = -5$ $\qquad a = 5$

deceleration is $5\ \text{m/s}^2$

10

(iv) $\text{distance} = \frac{1}{2}(5)(15)$
$+ (20)(15)$
$+ \frac{1}{2}(3)(15)$
$= 37.5 + 300 + 22.5$
$= 360\ \text{m}$

10

(v) $v^2 = u^2 + 2as$
$= 0 + 2(3)(13.5)$
$= 81$
$v = 9\ \text{m/s}$

10

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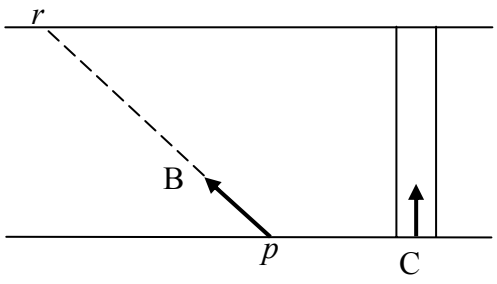
2. A river is 72 metres wide and has parallel banks.
A boat B departs from point *p* on the southern bank and lands at point *r* on the northern bank.



The actual velocity of B is $-4\vec{i}+3\vec{j}$ m/s.
Cyclist C travels due north at a constant speed of 4 m/s across a straight level bridge which spans the river.

Find

(i) the velocity of C in terms of $\vec{i}$ and $\vec{j}$

(ii) the velocity of B relative to C in terms of $\vec{i}$ and $\vec{j}$

(iii) the magnitude and direction of the velocity of B relative to C

(iv) the time it takes C to cross the river

(v) how much longer it will take B to cross the river.

(i) $\vec{V}_C = 0\,\vec{i} + 4\,\vec{j}$ — 10

(ii) $\vec{V}_{BC} = \vec{V}_B - \vec{V}_C$ — 5

$= \left(-4\,\vec{i}+3\,\vec{j}\right) - \left(0\,\vec{i}+4\,\vec{j}\right)$

$= -4\,\vec{i} - 1\,\vec{j}$ — 5

(iii) magnitude $= \sqrt{(-4)^2 + (-1)^2}$

$= \sqrt{17}$ or 4.12 m/s — 5

direction $= \tan^{-1}\left(\frac{1}{4}\right)$

or $14.04°$ with bank. — 5

(iv) time $= \frac{72}{4} = 18$ seconds — 10

(v) time $= \frac{72}{3} = 24$ seconds — 5

$\Rightarrow$ required time $= 6$ seconds — 5

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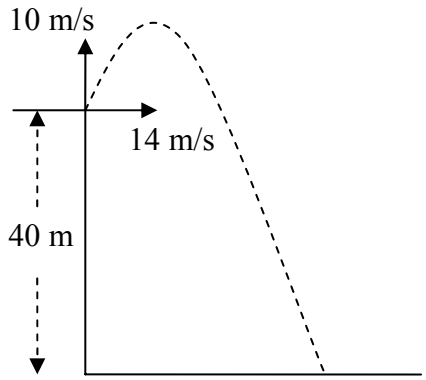
3. A projectile is fired with initial velocity $14\vec{i} + 10\vec{j}$ m/s from the top of a vertical cliff of height 40 m.



(i) Calculate the time taken to reach the maximum height.
(ii) Calculate the maximum height of the projectile above ground level.
(iii) Calculate the time it takes the projectile to travel from the maximum height to the ground.
(iv) Find the range.
(v) Find the speed of the projectile as it strikes the ground.

(i) $v_y = 0$ $\quad v = u + at$

$10 - 10t = 0$ $\quad 0 = 10 - 10t$

$t = 1$ s $\quad t = 1$ s — 10

(ii) maximum ht. $= \left(10t + \frac{1}{2}at^2\right) + 40$

$= 10(1) - 5(1)^2 + 40$

$= 45$ m — 10

(iii) $s = ut + \frac{1}{2}at^2$

$45 = 0 + 5t^2$

$t^2 = 9$

$t = 3$ s — 10

(iv) time $= 1 + 3 = 4$

range $= 14(4)$

$= 56$ m — 10

(v) $v^2 = u^2 + 2as$

$v^2 = 0 + 2(10)(45)$

$v^2 = 900$

$\Rightarrow v = 30$ m/s — 5

speed $= \sqrt{(14)^2 + (-30)^2}$

$= 33.11$ m/s — 5

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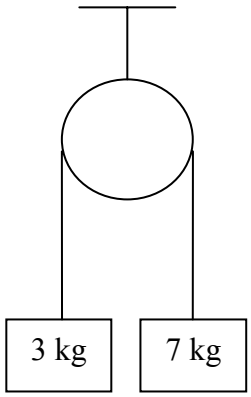
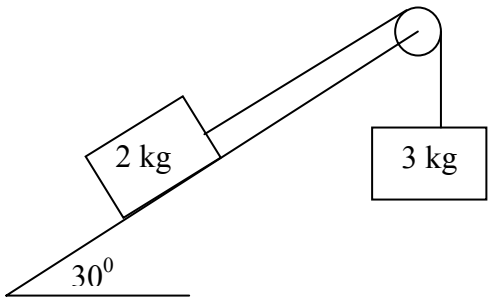
4. **(a)** Two particles of masses 7 kg and 3 kg are connected by a taut, light, inelastic string which passes over a smooth light pulley. The system is released from rest.



Find **(i)** the common acceleration of the particles.

(ii) the tension in the string.

(b) A rough plane is inclined at 30^0 to the horizontal and has a smooth light pulley attached to its uppermost point.
A taut, light, inelastic string passes over the pulley and has masses of 3 kg and 2 kg attached to its end points.
The coefficient of friction between the 2 kg mass and the plane is $\frac{1}{\sqrt{3}}$.



The 3 kg mass hangs vertically.
The system is released from rest.
The 3kg mass moves vertically downwards.

(i) Show on separate diagrams all the forces acting on each mass.
(ii) Find the common acceleration.
(iii) Find the tension in the string.

4 (a) (i)

$T - 3g = 3a$ 5

$7g - T = 7a$ 5

$a = \frac{40}{10} = 4 \text{ m/s}^2$ 5

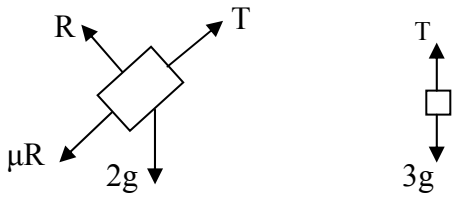
(ii)

$T - 3g = 3a$

$T - 30 = 12$

$T = 42 \text{ N}$ 5 20

4(b) (i)



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(ii) $3g - T = 3a$

$\Rightarrow T = 30 - 3a$

5

$R = 2g\cos 30$

$T - 2g\sin 30 - \mu R = 2a$

5

$$(30 - 3a) - 2g\left(\frac{1}{2}\right) - \left(\frac{1}{\sqrt{3}}\right)\left(g\sqrt{3}\right) = 2a$$

$30 - 3a - g - g = 2a$

$5a = 10$

$a = 2$

5

(iii) $T = 30 - 3a$

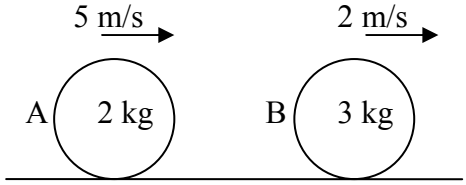
$= 30 - 6$

$= 24$ N

5

30

5. A smooth sphere A, of mass 2 kg, collides directly with another smooth sphere B, of mass 3 kg, on a smooth horizontal table.



A and B are moving in the same direction with speeds of 5 m/s and 2 m/s respectively.

The coefficient of restitution for the collision is $\frac{2}{3}$.

Find

(i) the speed of A and the speed of B after the collision

(ii) the loss in kinetic energy due to the collision

(iii) the magnitude of the impulse imparted to B due to the collision.

(i) PCM $2(5) + 3(2) = 2v_1 + 3(v_2)$

$16 = 2v_1 + 3v_2$ — 10

NEL $v_1 - v_2 = -e(u_1 - u_2)$

$= -\frac{2}{3}(5-2)$ — 10

$= -2$

$v_1 = 2$ m/s and $v_2 = 4$ m/s — 10

(ii) KE before collision $= \frac{1}{2}(2)(5)^2 + \frac{1}{2}(3)(2)^2$

$= 31$ — 5

KE after collision $= \frac{1}{2}(2)(2)^2 + \frac{1}{2}(3)(4)^2$

$= 28$ — 5

KE lost $= 31 - 28$

$= 3$ J — 5

(iii) Impulse $= |(3)(2) - (3)(4)|$

$= 6$ Ns — 5

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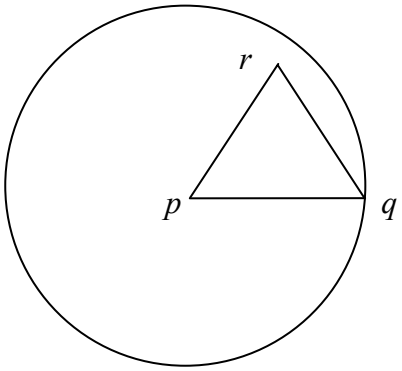
6. **(a)** Particles of weight 2 N, 3 N, 4 N and 5 N are placed at the points $(4,3)$, $(2,-3)$, $(-5,6)$ and $(4,-7)$, respectively.
Find the co-ordinates of the centre of gravity of the system.

(b) A circular lamina with centre p and with point q on its circumference has the triangular portion with vertices p, q and r removed.



The co-ordinates of the points are $p(0,0)$, $q(8,0)$ and $r(4,6)$ respectively.

Find the co-ordinates of the centre of gravity of the remaining lamina.

(a)
$$\bar{x} = \frac{2(4)+3(2)+4(-5)+5(4)}{14} \quad 10$$
$$\bar{x} = 1 \quad 5$$
$$\bar{y} = \frac{2(3)+3(-3)+4(6)+5(-7)}{14} \quad 10$$
$$\bar{y} = -1 \quad 5$$

(b)

	area :	c.g.	
pqr	$\frac{1}{2}(8)(6) = 24$	$(4, 2)$	5
lamina	$\pi(8)^2 = 64\pi$	$(0, 0)$	5
remainder	$: 64\pi - 24$	(x, y)	

$$(64\pi - 24)(x) = 64\pi(0) - 24(4)$$
$$x = -0.54 \quad 5$$

$$(64\pi - 24)(y) = 64\pi(0) - 24(2)$$
$$y = -0.27 \quad 5$$

co - ords of c.g. $(-0.54, -0.27)$

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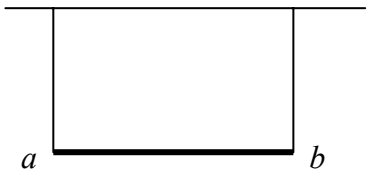
7. **(a)** A uniform beam, ab, is held in a horizontal position by two vertical inelastic strings attached at a and b respectively.

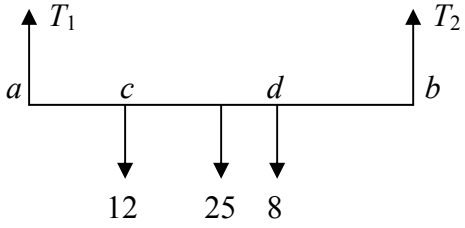


The weight of the beam is 25 N.
The length of the beam is 4 m.

A particle of weight 12 N is placed at a point c on the beam and a particle of weight 8 N is placed at a point d on the beam.

$|ac| = 1$ m and $|db| = 1.5$ m.

Calculate the tension in each of the strings.



Resolve forces $T_1 + T_2 = 12 + 25 + 8$ 5

$T_1 + T_2 = 45$

Take moments about a : 5

$T_2(4) = 12(1) + 25(2) + 8(2.5)$

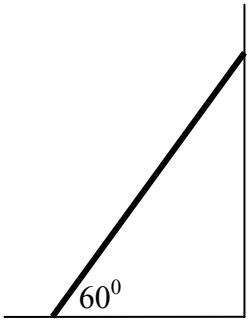
$4T_2 = 82$

$T_2 = 20.5$ N 5

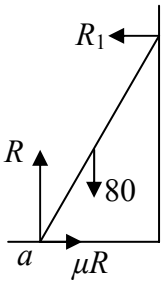
$T_1 = 45 - T_2$
$= 45 - 20.5$
$= 24.5$ N 5

25

7 (b) A uniform ladder rests on rough horizontal ground and leans against a smooth vertical wall.
The length of the ladder is 5 m and its weight is 80 N.
The angle between the ladder and the ground is 60^0.
The ladder is on the point of slipping.



(i) Show on a diagram all the forces acting on the ladder.
(ii) Calculate the value of the coefficient of friction



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vert $R = 80$ 5

horiz $R_1 = \mu R$ 5
$= 80\mu$

Take moments about a :

$R_1(5\sin 60) = 80(2.5\cos 60)$ 5

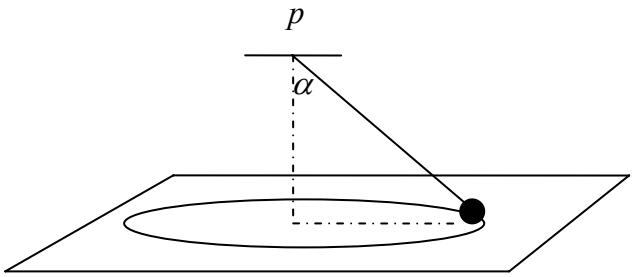
$80\mu(5)\left(\frac{\sqrt{3}}{2}\right) = 80(2.5)\left(\frac{1}{2}\right)$

$\mu = \frac{1}{2\sqrt{3}}$ 5 25

8. **(a)** A particle describes a horizontal circle of radius r m with uniform angular velocity ω radians per second.
Its speed and acceleration are 2 m/s and 4 m/s^2 respectively.
Find

(i) the value of r
(ii) the value of ω.

(b) A smooth particle of mass 2 kg is attached by a light inelastic string to a fixed point p.
The particle describes a horizontal circle of radius 0.5 m on the smooth surface of a horizontal table.



The centre of the circle is vertically below the point p.
The string makes an angle α with the vertical, where $\tan\alpha = \frac{3}{4}$.
The tension in the string is 15 Newtons.

Find

(i) the reaction force between the particle and the table
(ii) the angular speed of the particle.

(a)

$r\omega = 2$ — 10

$r\omega^2 = 4$ — 10

$\Rightarrow \omega(r\omega) = 4$

$\Rightarrow \omega(2) = 4$

$\Rightarrow \omega = 2$ rad/s — 5

$\Rightarrow r = 1$ m — 5

(b)

(*i*) $15\cos\alpha + R = 20$

$R = 20 - 15\left(\frac{4}{5}\right)$

$= 8$ N — 10

(*ii*) $15\sin\alpha = mr\omega^2$

$15\left(\frac{3}{5}\right) = 2\left(\frac{1}{2}\right)\omega^2$

$\Rightarrow \omega = 3$ rad/s — 10

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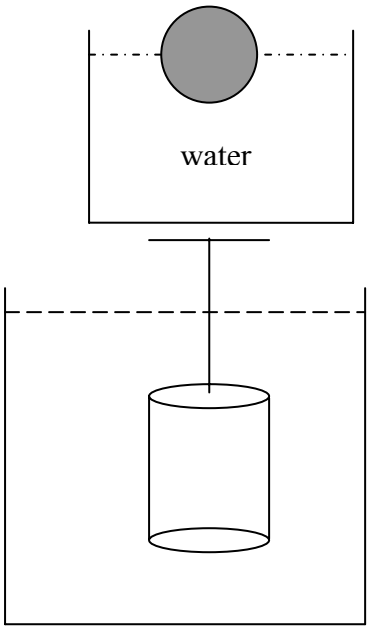
9. **(a)** A solid sphere floats at rest in water. The radius of the sphere is 7 cm. Half of the sphere lies below the surface of the water. Find, correct to one place of decimals, the weight of the sphere.



(b) A right circular solid cylinder has a height of 14 cm and a radius of 3 cm.

The relative density of the cylinder is 5 and it is completely immersed in a liquid of relative density 0.9. The cylinder is held at rest by a light inelastic string which is attached to a fixed support. The top of the cylinder is horizontal as shown in the diagram.

Find the tension in the string.

[Density of water = 1000 kg/m^3]

(a)

$$B = W \qquad 5$$

$$1000\left(\frac{V}{2}\right)g = \rho V g \qquad 5$$

$$\rho = 500 \qquad 5$$

$$W = \rho V g$$

$$= 500\left(\frac{4}{3}\pi(0.07)^3\right)(10) \qquad 5$$

$$= 7.2 \text{ N.} \qquad 5$$

(b)

$$T + \mathrm{B} = W \qquad 5$$

$$T + \frac{W s_L}{s} = W$$

$$T + \frac{W(0.9)}{5} = W \qquad 5$$

$$T = \frac{41W}{50} \qquad 5$$

$$= \frac{41}{50}\left\{5000\left(\pi(0.03)^2(0.14)\right)10\right\} \qquad 5$$

$$T = 16.236 \text{ N} \qquad 5$$

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